



***Nonlinear Multi-scale Modeling  
of Rubber and Tires with DIGIMAT***

[www.e-Xstream.com](http://www.e-Xstream.com)

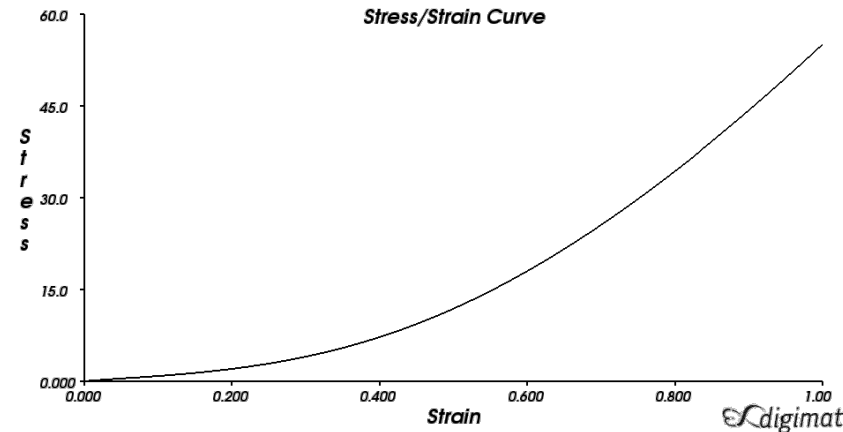
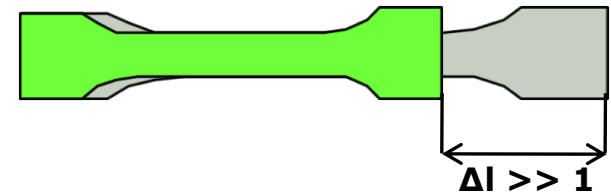
## ∞ Introduction

## ∞ DIGIMAT for Rubber Matrix Composites

- ✓ DIGIMAT-MF: Effect of Carbon Black Content on the Rubber Stiffness (stress-Strain)
- ✓ DIGIMAT-FE: Effect of Carbon Black Clustering on the Stress-Strain Response
- ✓ DIGIMAT to CAE: Effect of Carbon Black on Tire Deflection & Footprint

## ∞ Conclusions

- ∞ Large deformation
- ∞ Non linear stress-strain behavior
- ∞ (Quasi) Incompressible
- ∞ Applications
  - ✓ Tires
  - ✓ Anti-vibration systems
  - ✓ Seals
  - ✓ Hoses and fluid transport systems.
  - ✓ etc





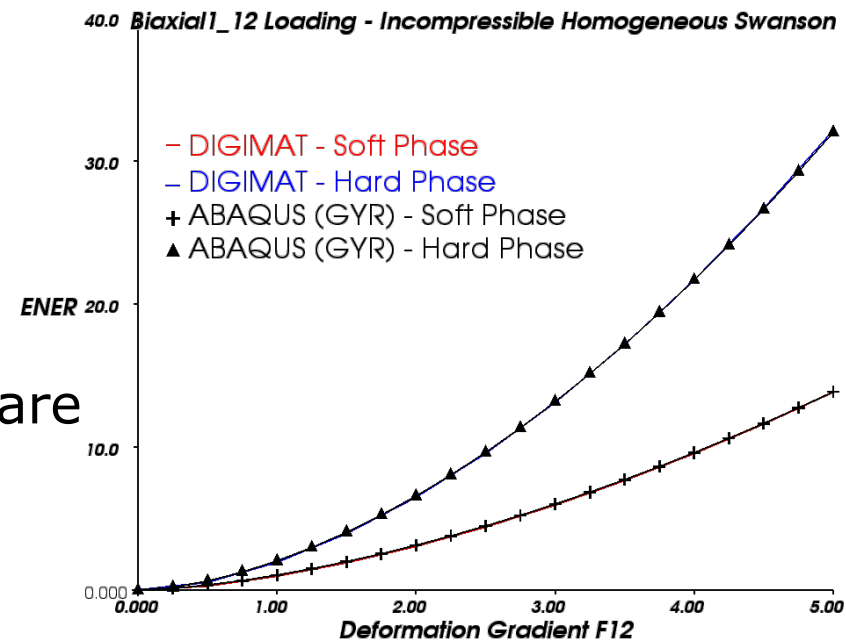
# Hyperelastic (Finite Strain) Material Behavior

## 5 (Micro) Hyperelastic Models

- ✓ neo-Hookean (Compressible/Incompressible)
- ✓ Mooney-Rivlin (Compressible/Incompressible)
- ✓ Ogden (Compressible/Incompressible)
- ✓ Swanson (Compressible/Incompressible)
- ✓ Storakers (Highly Compressible)

## Temperature-dependent Properties

## Verified against standard FEA Software





# Mean-Field Homogenization of RMC

## Mean Field Homogenization (Finite Strain)

- ✓ Mori-Tanaka
- ✓ Interpolative Double Inclusion

## 2 to N-Phase "Composite"

- ✓ Hyperelastic/Hyperelastic
- ✓ Hyperelastic/Elastic
- ✓ Hyperelastic/Rigid (Work in Progress)
- ✓ Elastic/Hyperelastic
- ✓ Hyperelastic/Void

## Microstructure Morphology

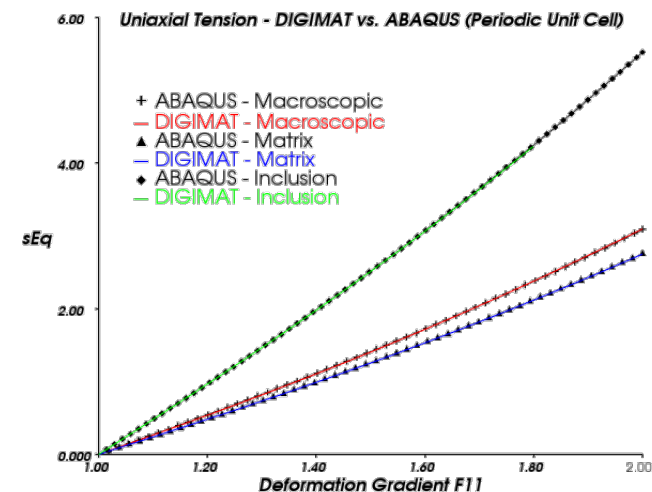
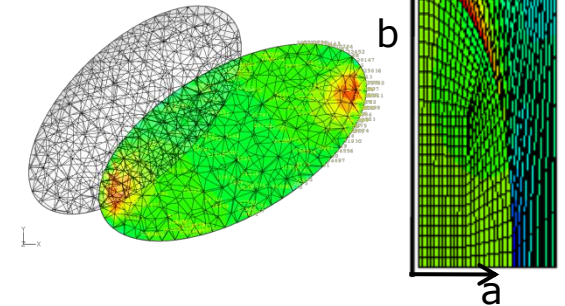
- ✓ Inclusion Orientation:
  - Fixed/Random/General (Orientation Tensor)
- ✓ Inclusion Shape
  - Platelets/Spheres/Fibers (short to continuous)
- ✓ Inclusion Volume Fraction
  - From Low to High

## Thermo-mechanical Loading

## Verified against Unit Cell FEA

Evolution of:

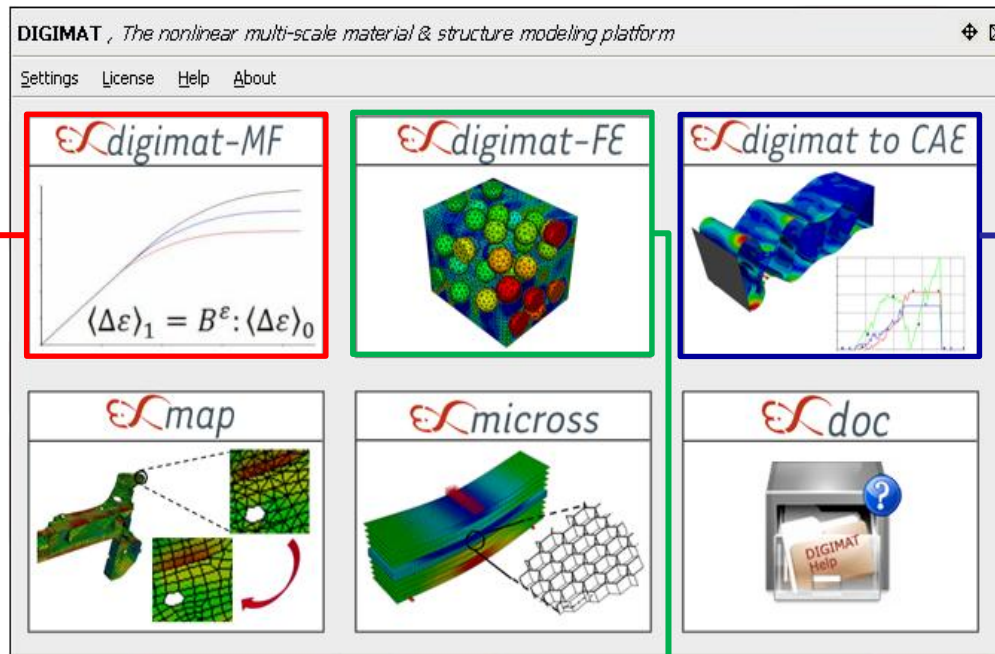
- Inclusions' shape
- Inclusions' orientation





# Application: Carbon Black Filled Rubber

Effect of Carbon Black Content on the Rubber Stiffness (stress-Strain)



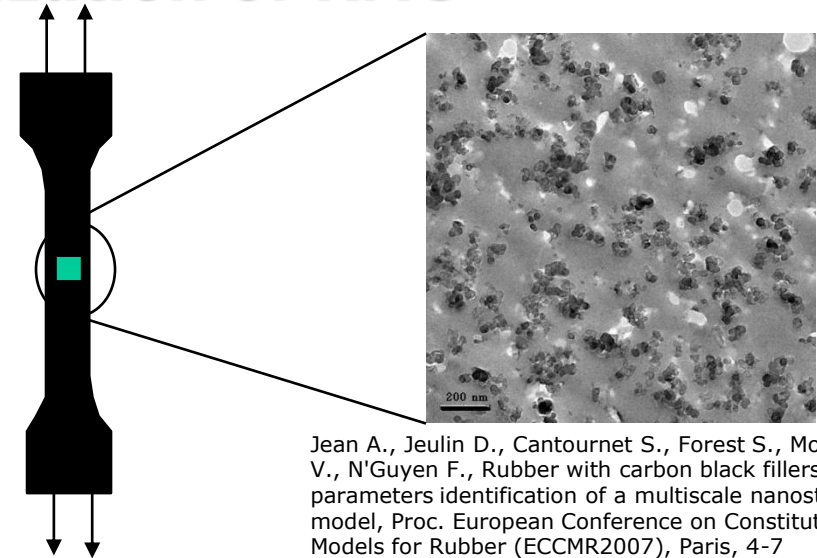
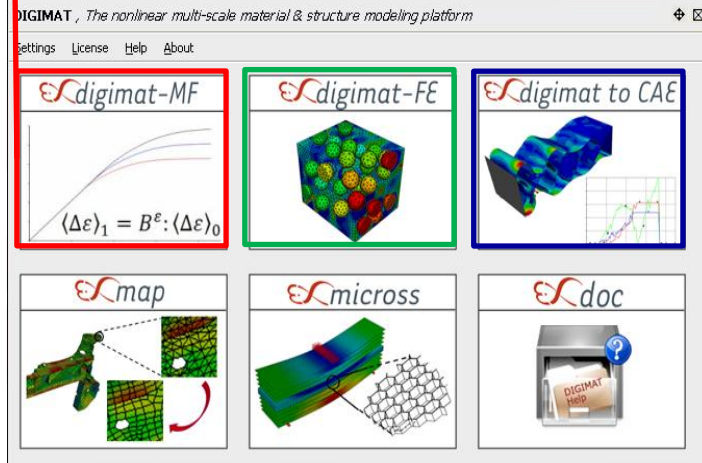
Effect of Carbon Black on Tire Deflection & Footprint

Effect of Carbon Black Clustering on the Stress-Strain Response



# Mean-Field Homogenization of RMC

## Effect of Carbon Black Content on the Rubber Stiffness (stress-Strain)



Jean A., Jeulin D., Cantournet S., Forest S., Mounoury V., N'Guyen F., Rubber with carbon black fillers: parameters identification of a multiscale nanostructure model, Proc. European Conference on Constitutive Models for Rubber (ECCMR2007), Paris, 4-7 September 2007

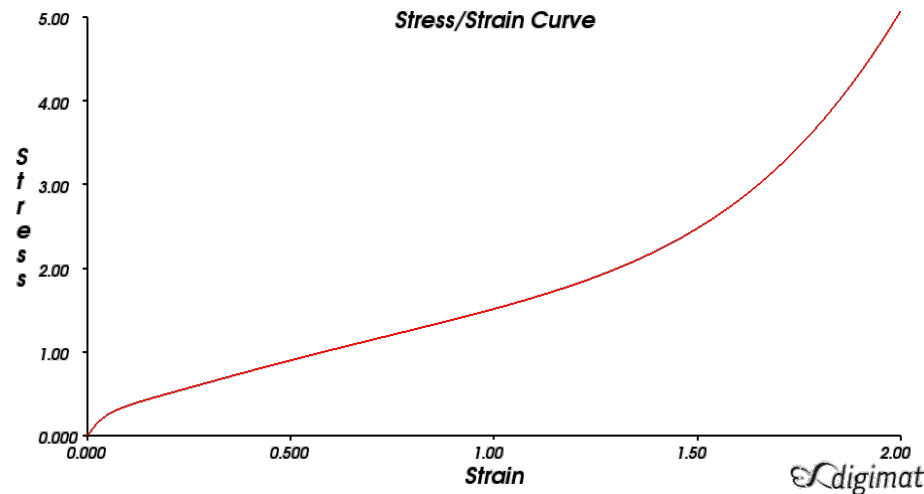
## ∞ Morphological analysis on the microstructure

### ∞ Matrix: Rubber

- ✓ Hyperelastic law : Ogden (N=3)  
(Coefficients fitted on experimental stress - strain curve)

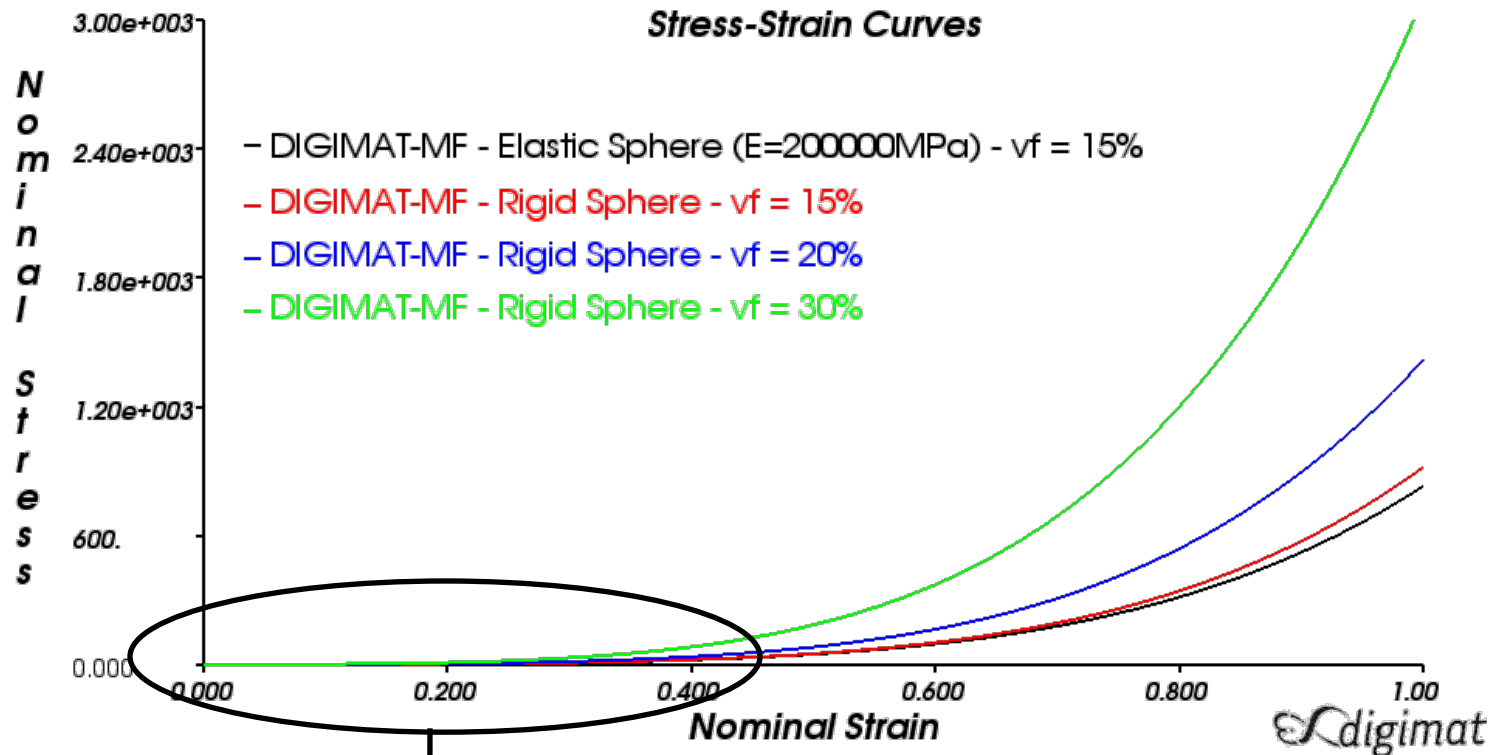
### ∞ Inclusion: Carbon Black

- ✓ Young Modulus = 200,000 MPa
- ✓ Poisson ratio = 0.3
- ✓ Volume fraction = 15%
- ✓ Aspect ratio = 1 (Sphere)





# Effect of Carbon Black Volume Fraction on S-S Curves



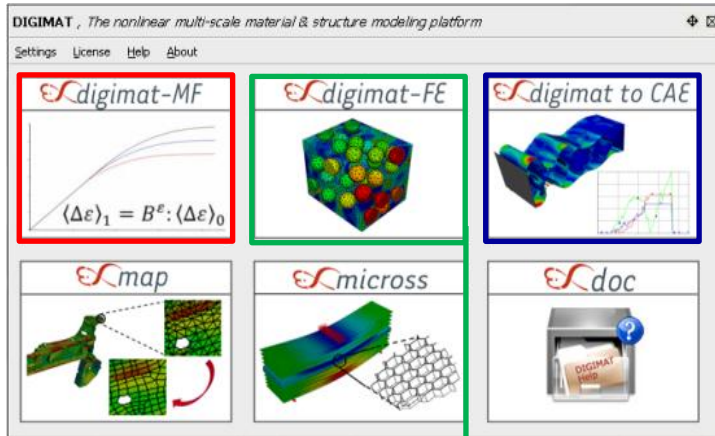
Rigid Formulation:

- More efficient for big contrast between the phases
- Faster in term of CPU Times (8/10 times faster)





# Carbon Black Clustering



## Effect of Carbon Black Clustering

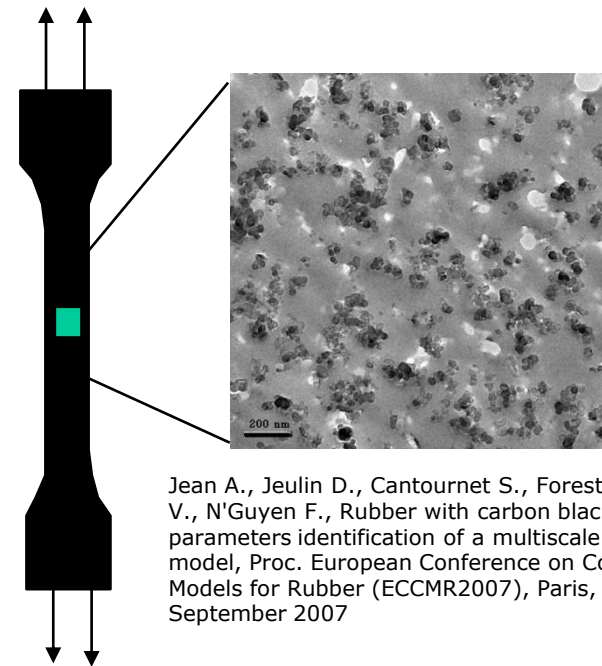
### ❑ Morphological Analyses of the real microstructure

- Phase distribution : clusters or random
- Inclusions' shape : Spheres (Assume to be of the same size)

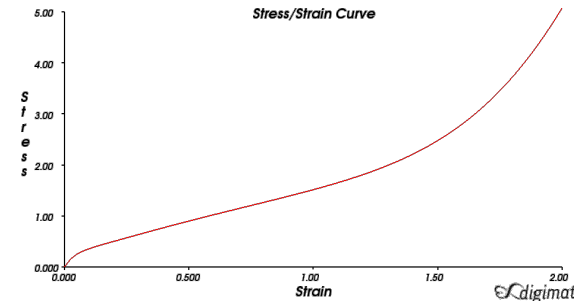
### ❑ Constituent properties

- Matrix: Rubber
  - ✓ Hyperelastic law : Ogden (N=3)
  - (Coefficients fitted on experimental stress – strain curve)

- Elastic Inclusions (Carbon Black) -> Big contrast between phase properties



Jean A., Jeulin D., Cantournet S., Forest S., Mounoury V., N'Guyen F., Rubber with carbon black fillers: parameters identification of a multiscale nanostructure model, Proc. European Conference on Constitutive Models for Rubber (ECCMR2007), Paris, 4-7 September 2007

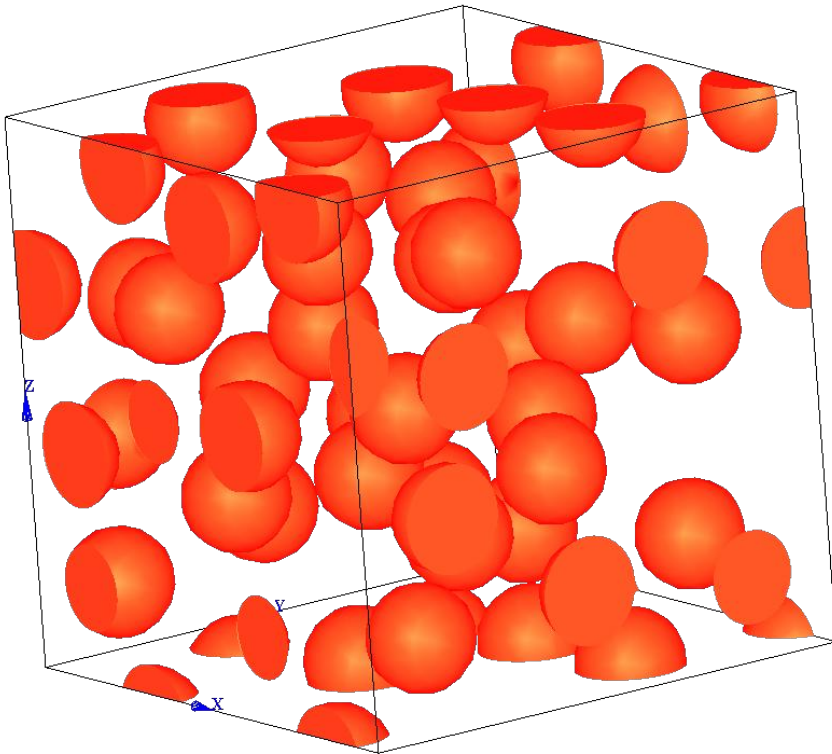


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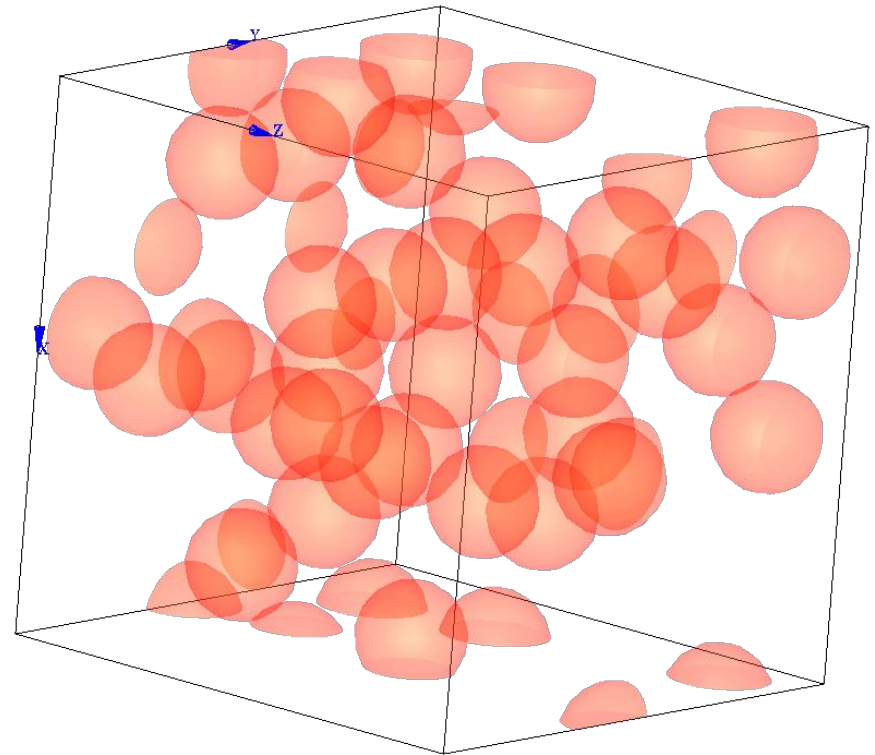


# Representative Volume Element Geometry

Microstructure without clusters  
(random distribution of inclusions)



Microstructure with Clusters



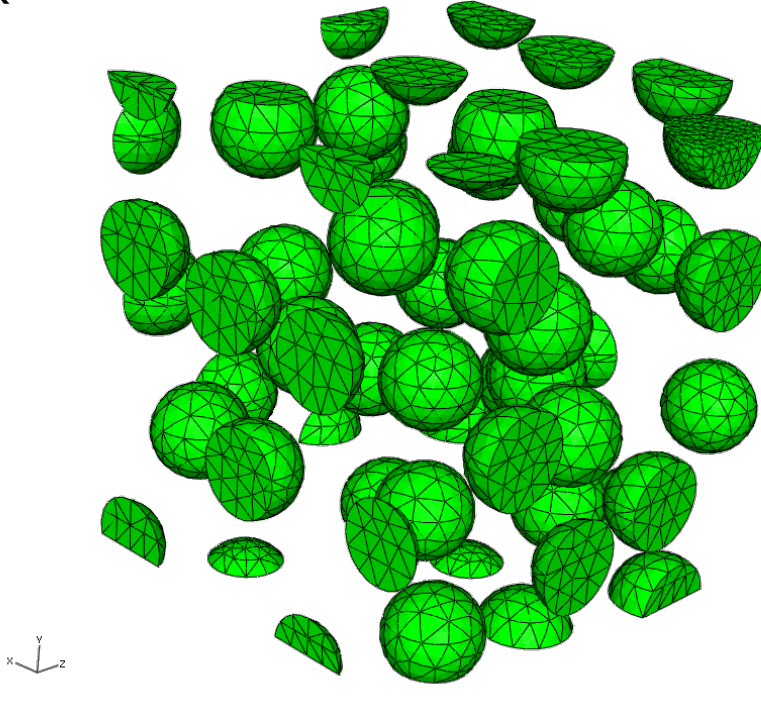
For both microstructures :

- Number of inclusions = 30
- Same size for each inclusion



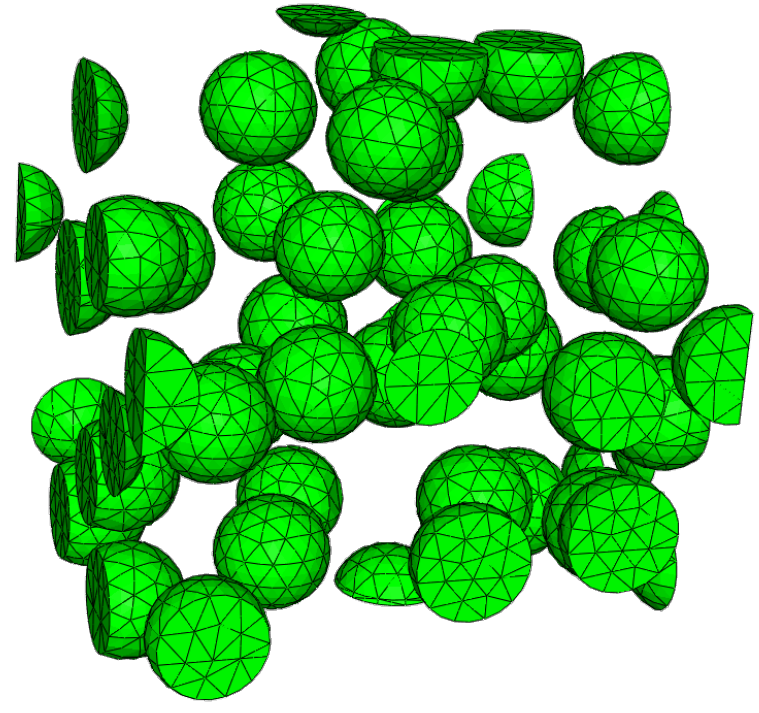
# Automatic RVE Meshing with DIGIMAT-FE to ABAQUS/CAE

Microstructure without clusters  
(random distribution of inclusions)



- Element Type:
  - C3D10M for Inclusion
  - C3D10MH for the matrix
- Number of Element = 66,917

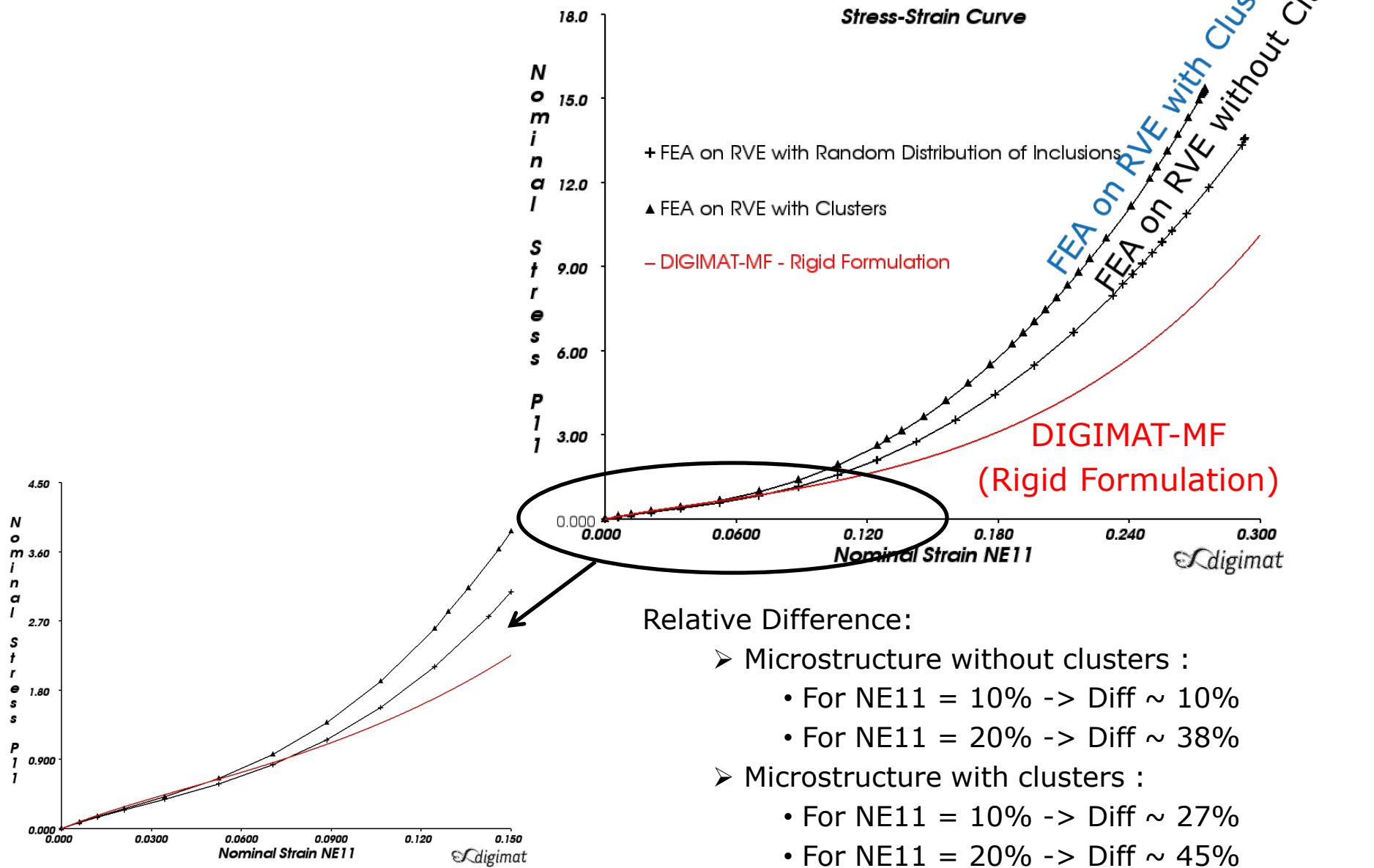
Microstructure with Clusters



- Element Type:
  - C3D10M for Inclusion
  - C3D10MH for the matrix
- Number of Element = 64,165



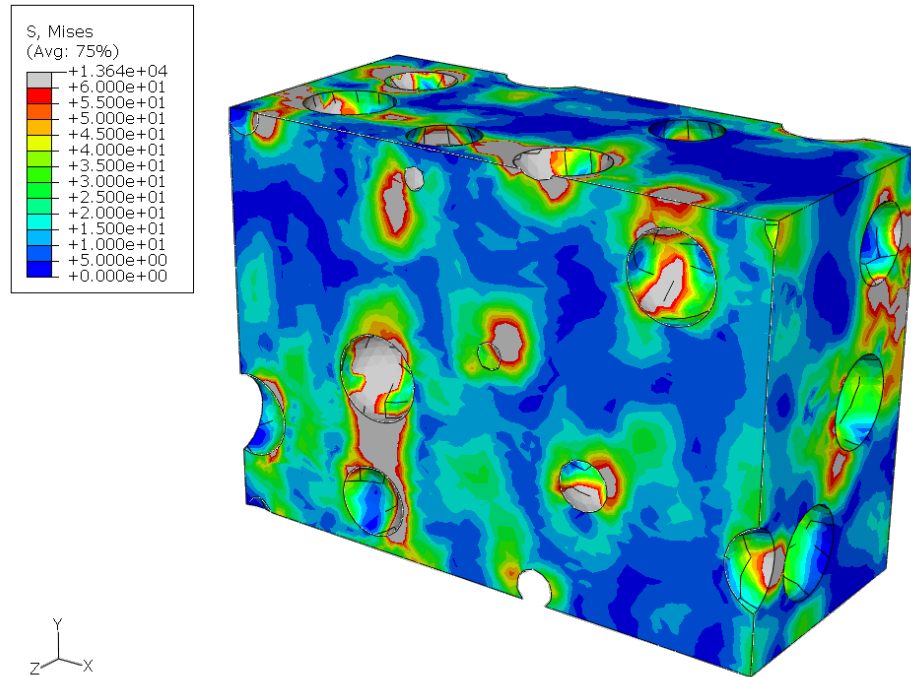
# FEA on RVE vs. DIGIMAT-MF



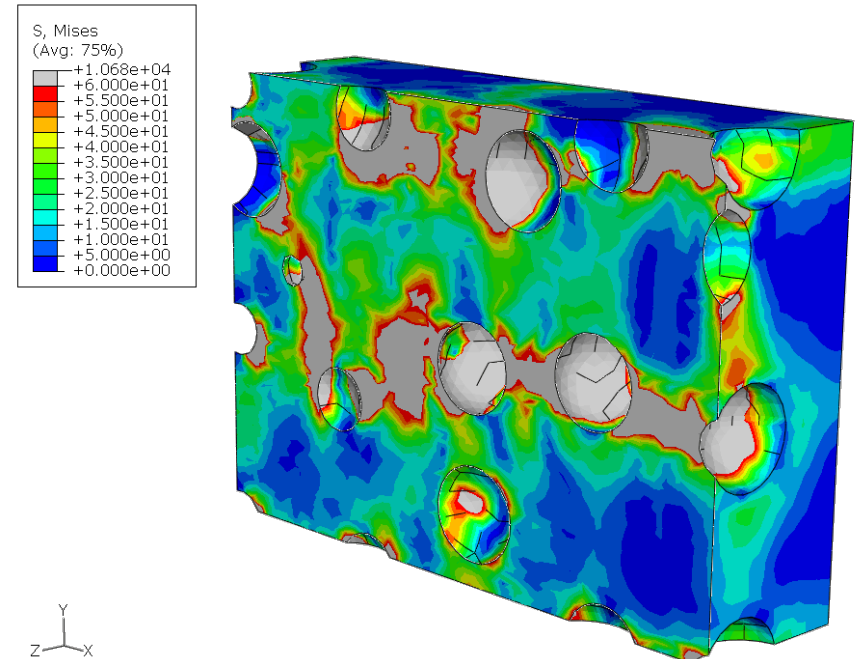


# Stress Distribution in the Rubber Matrix

Microstructure without Clusters  
(random distribution of inclusions)



Microstructure with Clusters



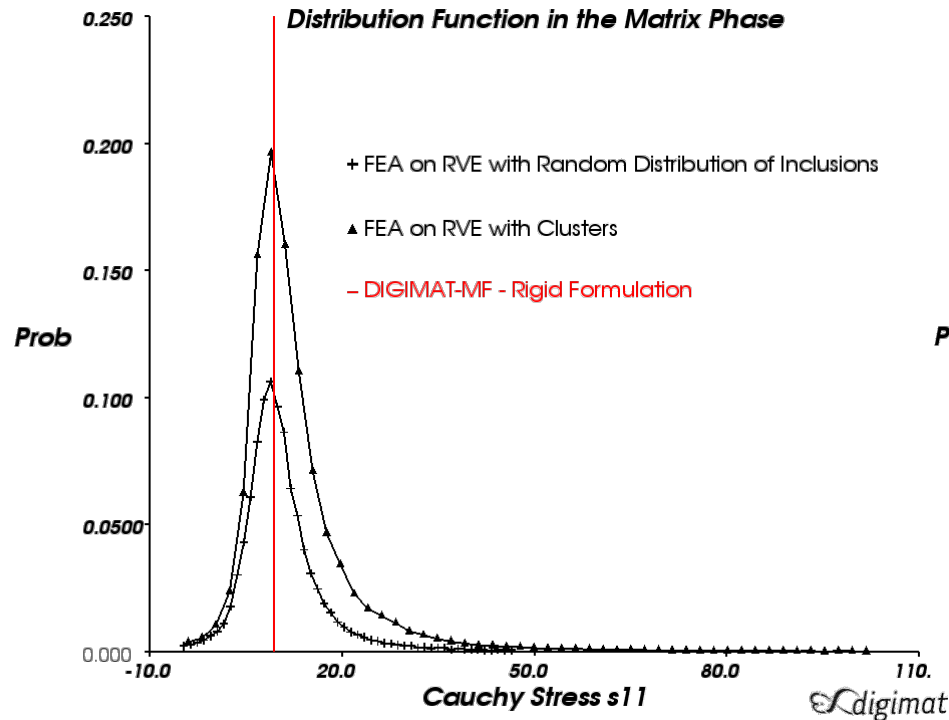
*Snapshot of local von-Mises stress for a nominal strain of 27%*

Difference on Maximum Stress between microstructure with clusters and microstructure without clusters = 60%



# Local Fields Distribution in the Matrix Phase

## Cauchy Stress Tensor



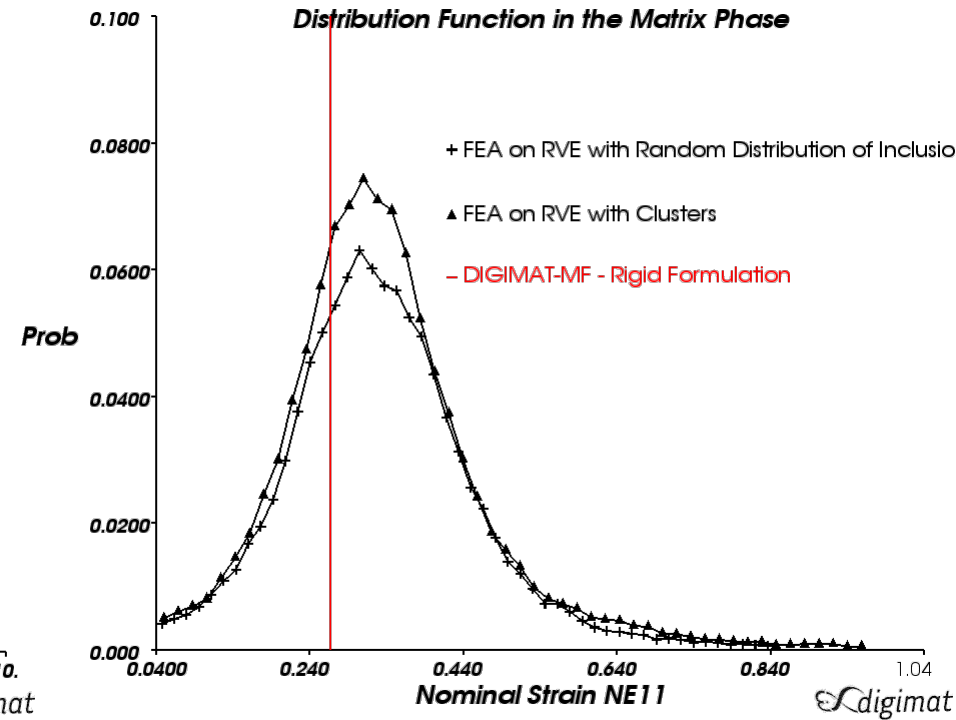
Microstructure without clusters :

- Min stress = -4.67 MPa
- Max stress = 46.43 MPa

Microstructure with clusters :

- Min stress = -3.97 MPa
- Max stress = 101.75 MPa

## Nominal Strain Tensor



Microstructure without clusters :

- Min strain = 4.73%
- Max strain = 83.63%

Microstructure with clusters :

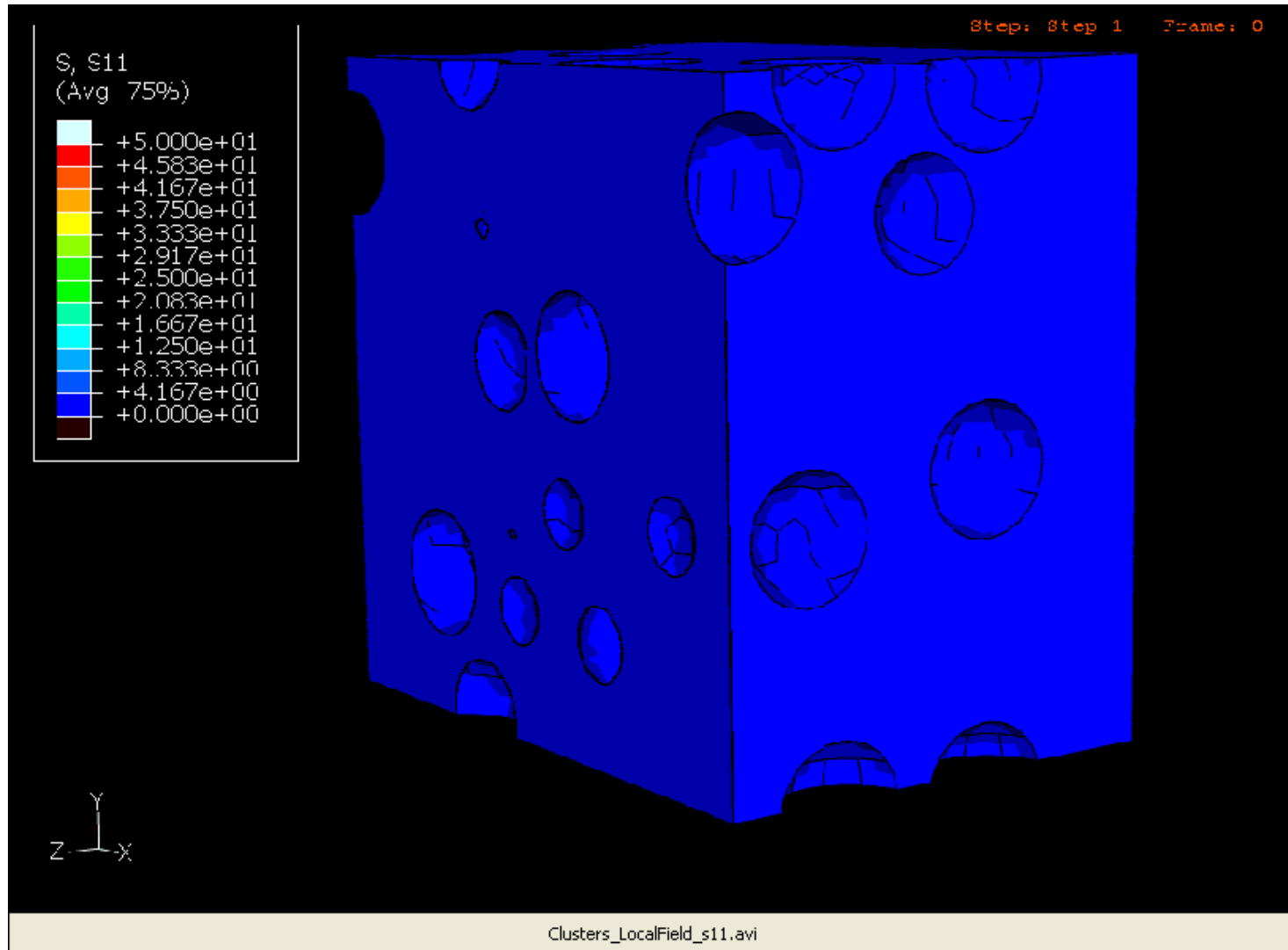
- Min strain = 5.02%
- Max strain = 95.87%

*Distribution function for a macroscopic nominal strain = 27%*





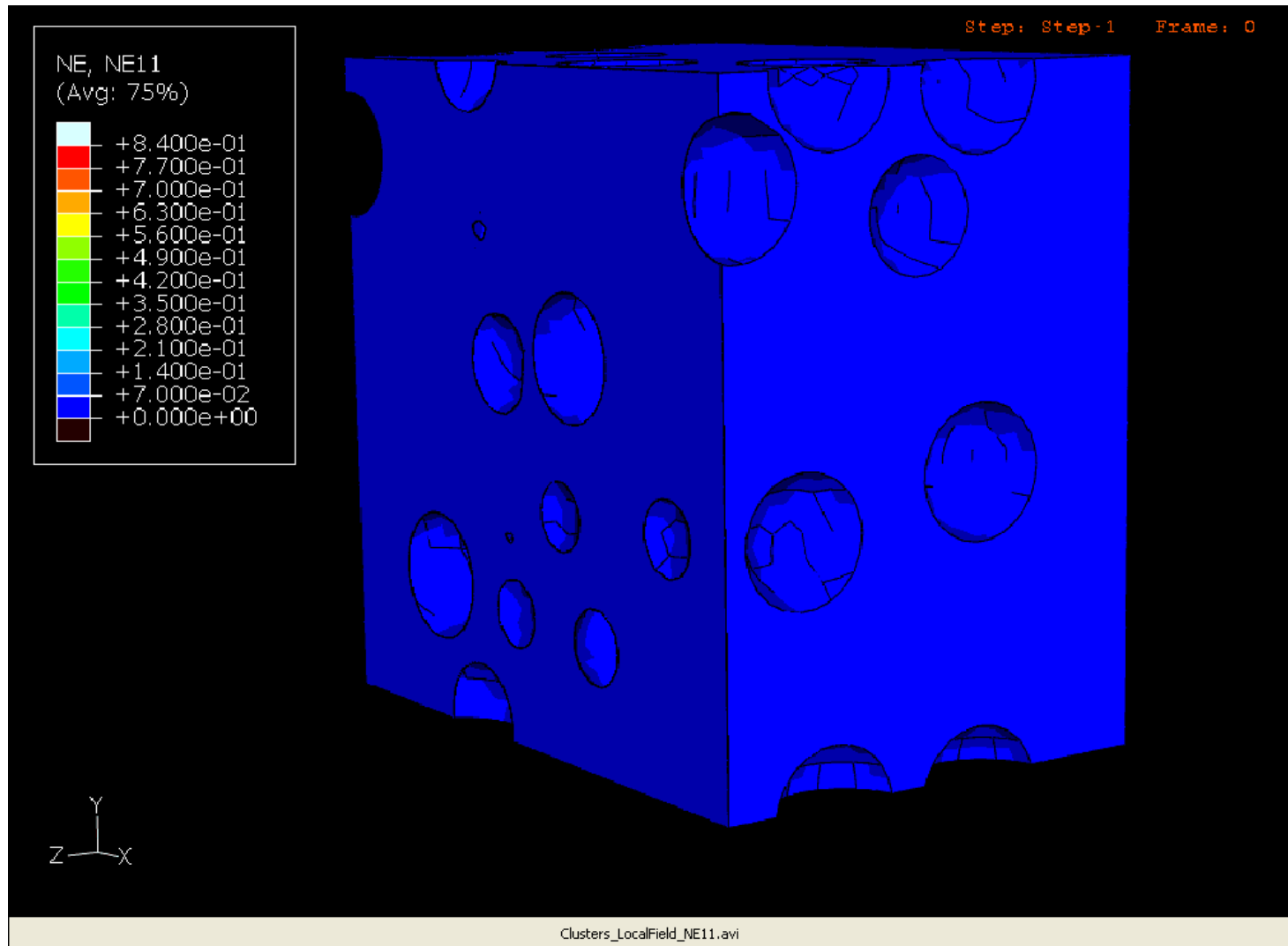
# Evolution of Stress/Strain in the Matrix Phase during the loading (1)



Cauchy Stress Tensor



# Evolution of Stress/Strain in the Matrix Phase during the loading (2)

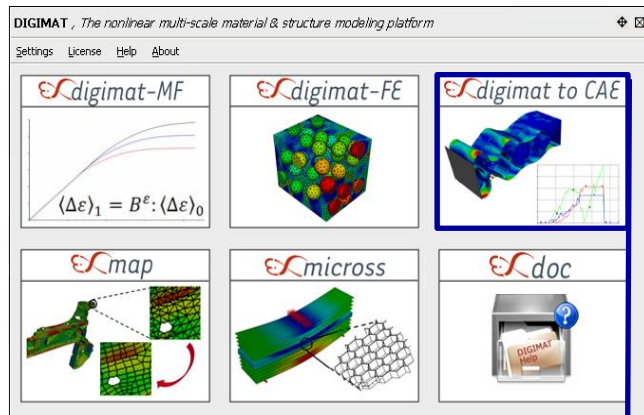


Nominal Strain tensor

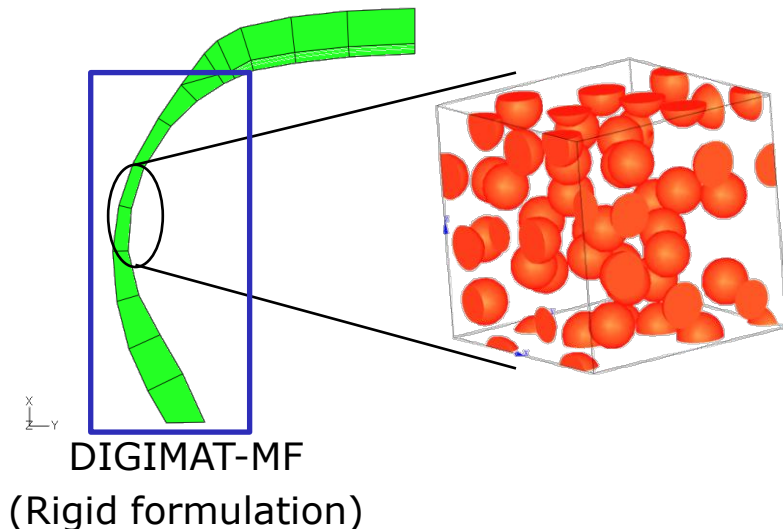




# Effect to Carbon Black on Tire Deflection & Footprint



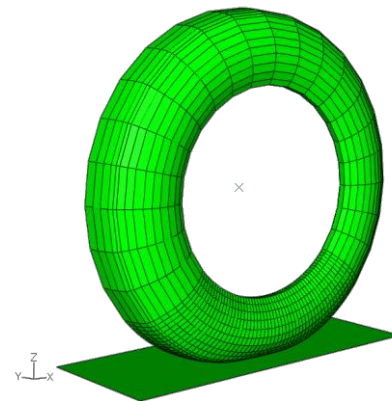
**Effect of Carbon Black  
Content on Tire Deflection  
& Footprint**



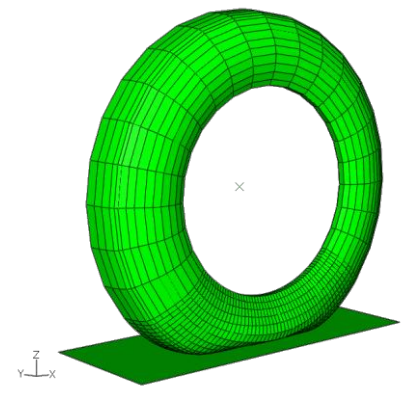
➡ Use DIGIMAT to CAE:

- Morphology effect on the global/Local response of a tire model under two different configurations:

Vehicle loading = 1640 N



Inflation Mode  
Pressure = 2 Bars



Deflation Mode  
Pressure = 0 Bar

Source: Abaqus Users' Manual

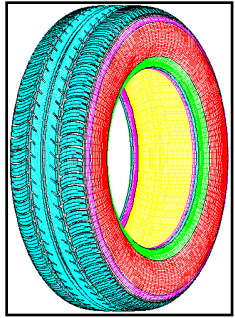
FEA of the Tire are performed:

- on a Linux 64 bits machine
- on a single processor



# Interaction between DIGIMAT and FEA

## Classical FE process



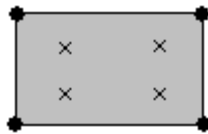
**FE model level**

Nodal coordinates, ...

Internal forces and  
element stiffness



**Element level**

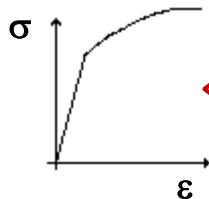


Strain increments,  
material state, ...

Stresses and  
material stiffness

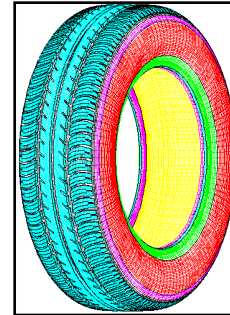


**Material level**



« In code » model

## Coupled FE/DIGIMAT process



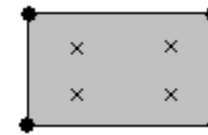
**FE model level**

Nodal coordinates, ...

Internal forces and  
element stiffness



**Element level**

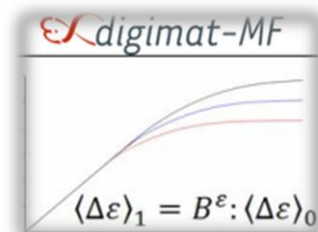


Strain increments,  
material state, ...

Stresses and  
material stiffness



**Material level**

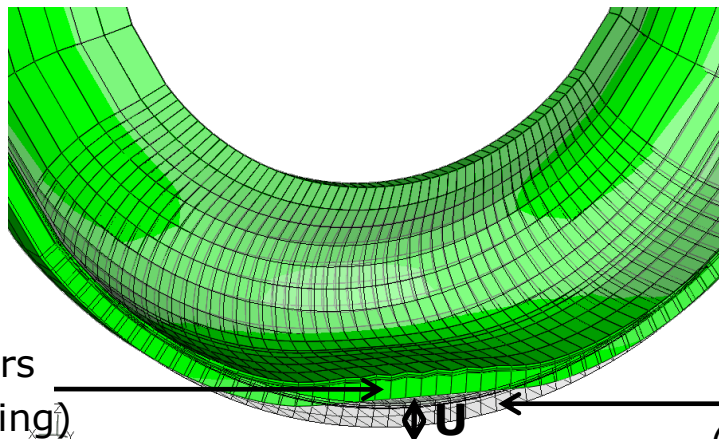




# Tire Deflection & Footprint @ 2 bars

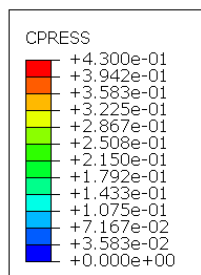
Homogeneous Matrix

- $U=19.3$  mm
- CPU=132 s



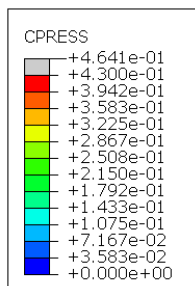
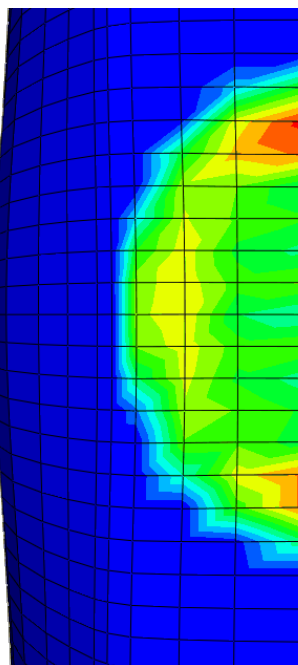
Pressure = 2 Bars  
(With vehicle loading)

Pressure = 2 Bars  
(Without vehicle loading)



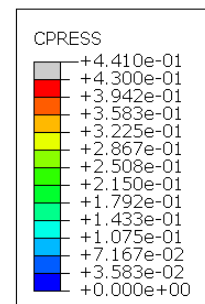
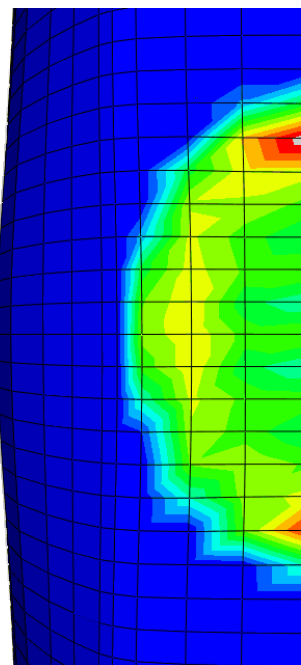
vf=15%

$U=19.82$  mm



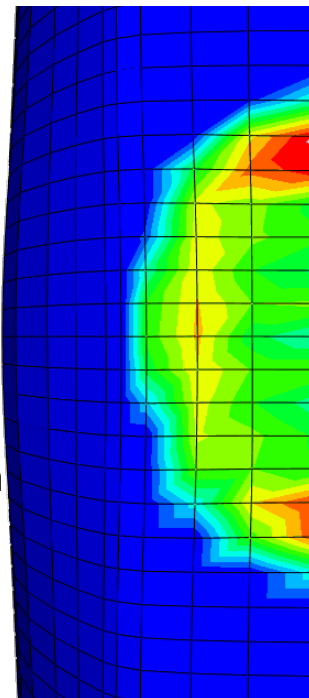
vf=20%

$U=19.83$  mm



vf=30%

$U=19.85$  mm

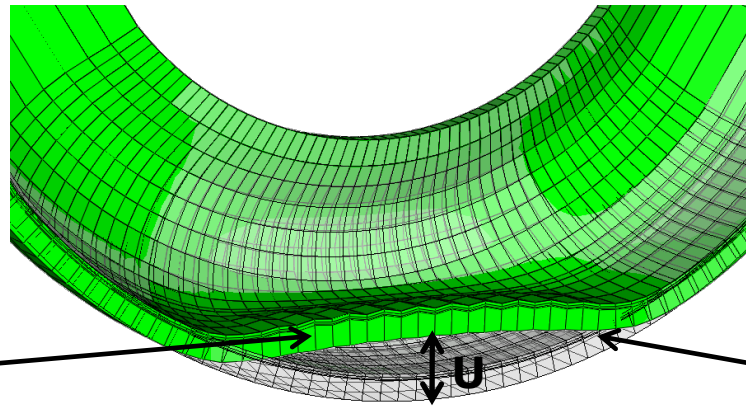




# Tire Deflection & Footprint @ 0 bar

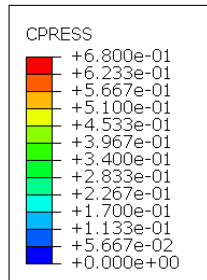
Homogeneous Matrix

- $U=51.55$  mm
- CPU=132 s



Pressure = 0 Bar  
(With vehicle loading)

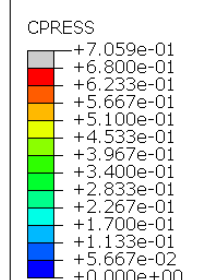
Pressure = 2 Bars  
(Without vehicle loading)



vf=15%

$U=44.38$  mm

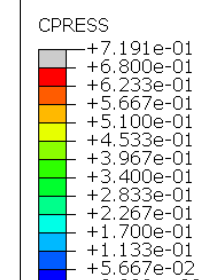
CPU=841.44 s



vf=20%

$U=41.27$  mm

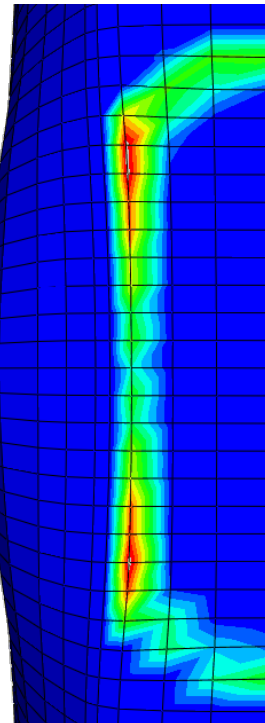
CPU=1293 s



vf= 30%

$U=34.26$  mm

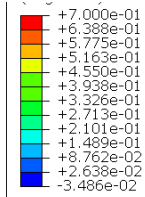
CPU=1193 s



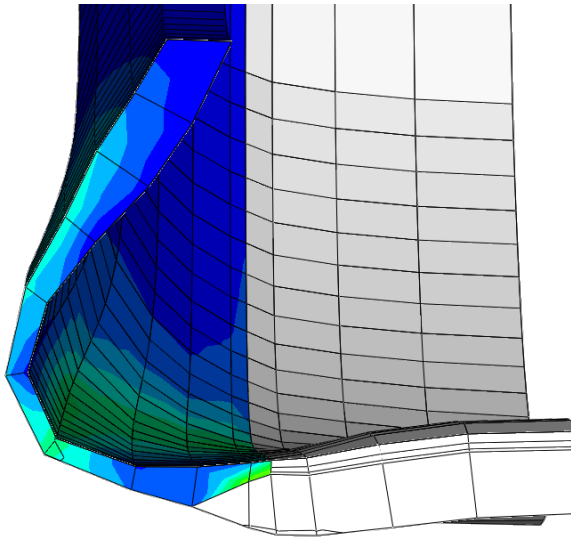


# Strain Energy in the Rubber Matrix

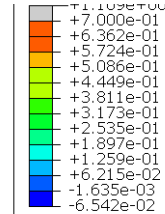
Strain  
Energy



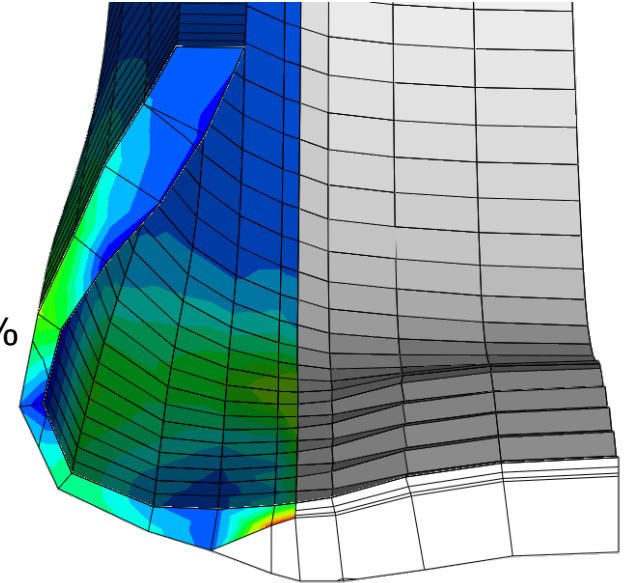
CB VF=15%



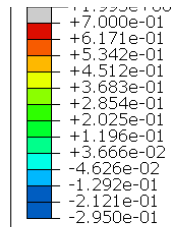
Strain  
Energy



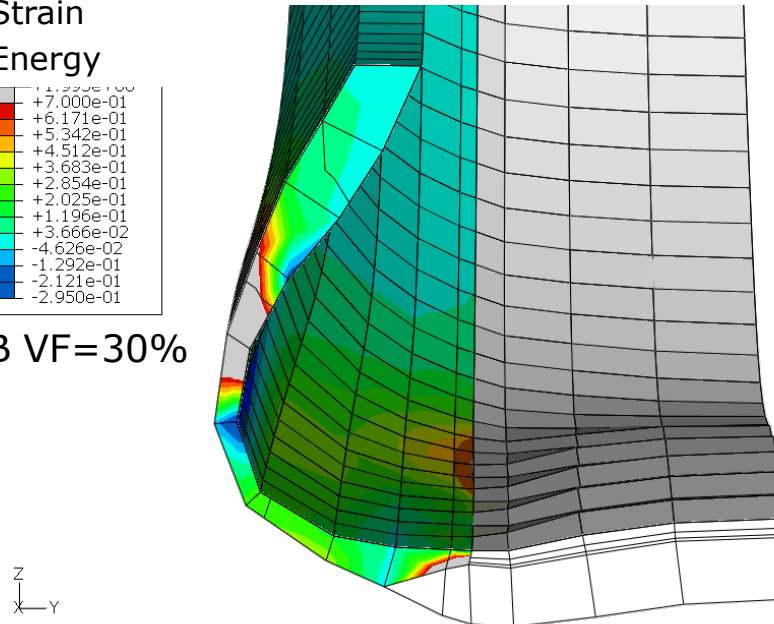
CB VF = 20%



Strain  
Energy



CB VF=30%



## ∞ DIGIMAT-MF

- ✓ Computes the nonlinear behavior of RMC as a function of the underlying microstructures
- ✓ A “RIGID” formulation was developed to better deal with very stiff inclusions
  - Minimizes CPU costs & Improves Convergence
  - Limited to spherical inclusions. The extension to fibers is ongoing
- ✓ Limited to random distribution of inclusions

## ∞ DIGIMAT-FE

- ✓ Predicts detailed information in the microstructure
- ✓ Take into account the exact inclusion distribution (e.g. clustering)

## ∞ DIGIMAT to CAE

- ✓ Use Mean-Field homogenization at the microscale
- ✓ Enables a strong coupling between the material microstructure and the behavior of the rubber structure

Acknowledgments: e-Xstream would like to thank Goodyear and the Luxembourg Ministry of Economy for supporting the R&D work performed in the field of nonlinear mean-field homogenization of Rubber Matrix Composites.